Centrality Measure in Weighted Networks Based on an Amoeboid Algorithm

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Abstract
Within network analysis, various measures of node centrality have been developed. In this paper, for weighted networks, a new centrality measure based on an amoeboid algorithm is proposed, which is called Physarum centrality. The measure not only focuses on shortest paths, but also includes contributions from competing paths. With the amoeboid algorithm, each edge is endowed with a flux and edges on shorter paths are of greater flux. By defining flux as criticality of edges, Physarum centrality of a node can be easily calculated as summing up the criticality of edges attached to it. Examples and applications are given to show the effective of our proposed measure in weighted networks.

Keywords: Physarum Centrality; Centrality Measures; Amoeboid Algorithm; Weighted Networks

1 Introduction

Network, as a set of nodes with edges connecting them, is ubiquitous in the world, such as the Internet, transportation network, social network, biological network and so on [1]. Over the years, centrality measure, identifying the centrality of nodes, has been an essential tool in network analysis [2, 3, 4]. Many researchers have studied it in various applications [5, 6, 7, 8]. Three best known measures were developed for binary networks [2], namely, degree, closeness, betweenness. However, as many networks are intrinsically weighted and edge weights contain many useful information, many researchers have paid their attention to weighted networks [9, 10, 11]. Among

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the proposed centrality measures, most are based on the assumption that information only spreads through shortest paths, which is regarded unrealistic [12, 4].

Thus, in this paper, a new measure based on an amoeboid algorithm is proposed for weighted networks, which might be called \textit{Physarum centrality}, $C_P$. By including contributions from competing paths, it relaxes the assumption about shortest paths. What’s more, as the paths are shorter, the scores for the edges will be higher. Thus, an amoeboid algorithm inspired by the path finding process of \textit{Physarum polycephalum} is adopted. \textit{Physarum polycephalum}, as an amoeboid organism, can form a dynamic tubular network connecting discovered food sources. What’s more, it is capable of solving many graph theoretical problems [13, 14, 15] and the network it generates is of high intelligence and performance [16]. The model inspired by \textit{Physarum} [17] can find optimal paths (not only shortest paths) connecting two specified nodes and shorter paths are with larger flux, which is fit for our centrality measure.

The outline of this paper is as follows. Section 2 begins with a brief introduction of \textit{Physarum} model for path finding in weighted networks. In Section 3, we define the proposed \textit{Physarum} centrality measure and show how it is calculated. Examples and applications of our measure are illustrated in Section 4. And in Section 5, we give our conclusions.

\section{Physarum Model for Path Finding [17]}

Assume the shape of \textit{Physarum} is represented as an undirected weighted network. Edges weights are denoted as the length of each edge. And the starting node $s$ and ending node $t$ in path finding act as two food sources for \textit{Physarum}. The variable $Q_{ij}$ denotes the flux through the edge $e_{ij}$ between node $i$ and $j$. Assuming approximate Poiseuille flow, the flux is given as:

$$Q_{ij} = \frac{D_{ij}}{L_{ij}} (p_i - p_j)$$

where $p_i$ is the pressure at node $i$, $L_{ij}$ is the length of edge $e_{ij}$ and $D_{ij}$ is its conductivity which is assigned with a value that belongs to $(0, 1]$ in the initialization.

By considering the balance of flux through each node, we have $\sum_i Q_{ij} = 0$ ($j \neq s, t$), $\sum_i Q_{is} + I_0 = 0$ and $\sum_i Q_{it} - I_0 = 0$, where $I_0$ is the flux from node $s$ which is constant in the model.

With the equations above, the network Poisson equation for the pressure is derived:

$$\sum_i \frac{D_{ij}}{L_{ij}} (p_i - p_j) = \begin{cases} 
-I_0, & \text{for } j = s, \\
+I_0, & \text{for } j = t, \\
0, & \text{otherwise}. 
\end{cases}$$

(2)

By setting $p_2 = 0$ as a basic pressure level, all $p_i$’s can be calculated by solving equation Eq. 2, and each $Q_{ij}$ is also obtained by Eq. 1.

Next step is the adaptation for conductivity:

$$\frac{d}{dt} D_{ij} = f(|Q_{ij}|) - \alpha D_{ij}$$

(3)

where $\alpha$ is a decay rate of the tube. $f(Q)$ is an increasing function with $f(0) = 0$. More detailed description of $f(Q)$ can be found in [17].
Until now, one iteration has been finished. Next is to judge whether the termination criterion is met or not. If the specified criterion is fulfilled, edges without flux are cut off while others compose optimal paths found by *Physarum* model. Otherwise, the next iteration is executed by updating pressure at each node, as depicted in Fig. 1.

![Flow chart of Physarum model for path finding](image)

Fig. 1: The flow chart of *Physarum* model for path finding

### 3 *Physarum* Centrality

In weighted networks, degree centrality has been extended as [9]:

\[
C_D^w (i) = \sum_j w_{ij}
\]  

(4)

However, it does not consider the global structure of the network. Although a node can be with high total edge weights, it might not access resources more quickly. To make up this limitation, our *Physarum* centrality of a node *C_P* (*i*) is defined as the sum of the criticality of each edge attached to it:

\[
C_P (i) = \sum_j c_{ij}
\]  

(5)

where *c_ij* denotes the criticality of edge *e_ij* between node *i* and *j*. The value of *c_ij* is derived by using *Physarum* model for path finding between all pairs of nodes in weighted networks.

For any pair of nodes *s* and *t*, *Physarum* model can find optimal paths between nodes *s* and *t*, by adapting the flux through each edge and its conductivity. When the adaptation is finished, optimal paths are reserved while others are delimitated as no flux through them. What’s more, the result shows that the shorter the path is, the greater the flux through it.
To capture this feature, the criticality of an edge can be defined as the sum of flux through the edge by applying Physarum model between all pairs of nodes:

\[ c_{ij} = \sum_k Q^k_{ij}, \quad k = 1, 2, 3, \ldots, \frac{n(n-1)}{2} \tag{6} \]

where \( Q^k_{ij} \) denotes the \( k^{th} \) final flux through edge \( e_{ij} \) by using Physarum model, while different \( k \) implies different path finding processes between different pairs of \( s \) and \( t \).

To summarize, the procedure for calculating our Physarum centrality is described as follows.

**Step 1** Construct the weight matrix \( W \). Since weights in most weighted networks stand for tie strengths and not the cost of them, the edge weights need to be reversed when constructing \( W \), in order to associate with the tube length in Physarum model.

**Step 2** Apply Physarum model to find optimal paths between all pairs of start node \( s \) and target node \( t \) and record the flux through each edge \( Q^k_{ij} \) in the \( k^{th} \) final result of the model. Before using the model, some configuration should be stated.

1. In the initialization, the length of each tube \( L_{ij} \) is initialized as weight in matrix \( W \) respectively and its conductivity \( D_{ij} \) is assigned with 0.5;
2. \( f(Q) \) in Eqn. 3 adopts \( f(Q) = (1 + a) Q^\mu / 1 + aQ^\mu \) with parameters set as \( a = 2, \mu = 27 \);
3. Termination criterion of the model is specified as maximum iterations, which is: \( 4 \log n \).

**Step 3** Calculate the criticality of each edge using Eqn. 6 with recorded values \( Q^k_{ij} \).

**Step 4** Calculate Physarum centrality with Eqn. 5.

### 4 Examples and Applications

In this section, examples and applications of our Physarum centrality measure in network analysis are given. And with comparison with another two measures: extended degree centrality [9] and flow betweenness centrality [12], Physarum centrality is shown as more reasonable.

#### 4.1 Simple Example

First, consider a simple weighted network based on hypothetical social proximities, which is adopted from [12] as shown in Fig. 2. For \( s = 1 \) and \( t = 2 \), there are three paths connecting them, among which path ‘1 → 2’ is the shortest, whose length is 0.3333 \( \left( \frac{1}{3} \right) \), with the largest flux through \( e_{12} \). The other two paths ‘1 → 3 → 2’ and ‘1 → 4 → 3 → 2’ are of equal length \( \left( 1 + \frac{1}{3} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} \right) \) and the flux through each path is equal and much smaller than path ‘1 → 2’.

Table 1 shows the results comparing our Physarum centrality \( C_P \) with extended degree centrality \( C_D^w \) and flow betweenness centrality \( C_F \). In the second column, \( C_D^w(1) = C_D^w(2) \), as the total edge weights of each node are equal. However, the edges connecting to node 2 have larger edge weights (shorter lengths) than that of others, which means \( e_{12} \) and \( e_{23} \) are more critical than other
Fig. 2: A simple weighted network

Table 1: Comparison results of three different centrality measures

<table>
<thead>
<tr>
<th>Node</th>
<th>Degree centrality $C_D$</th>
<th>Flow betweenness centrality $C_F$</th>
<th>Physarum centrality $C_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>0.35</td>
<td>6.0671</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0.25</td>
<td>7.1221</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0.65</td>
<td>11.5340</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.25</td>
<td>5.1619</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>0.00</td>
<td>4.0000</td>
</tr>
</tbody>
</table>

edges. Therefore, node 2 should be more central than node 1, as the result of our proposed $C_P$: $C_P(2) = 7.1221 > C_P(1) = 6.0671$. According to the third column in Table 1, the result of flow betweenness centrality is $C_F(3) > C_F(1) > C_F(2) = C_F(4) > C_F(5)$. However, it is obvious that the centrality of node 2 should be larger than that of node 4. Thus, the proposed Physarum centrality is more reasonable, as its result depicts: $C_P(3) > C_P(2) > C_P(1) > C_P(4) > C_P(5)$.

4.2 Application in Social Network

To illustrate the effect of the proposed Physarum centrality, an application is given in a social network among students living in a fraternity at a West Virginia college [18, 19]. The dataset BKFRAT includes 58 students and the number of times that a pair of students were seen in conversation is recorded as edge weights.

Table 2 demonstrates the result of three different centrality. According to Table 2, the most

Fig. 3: Ego networks of node 7, 2, 28 from social network dataset BKFRAT. The width of an edge corresponds to the number of social interactions between two individuals.
Table 2: Ranking of nodes according to three different centrality measures

<table>
<thead>
<tr>
<th>Rank</th>
<th>$C_D^w$</th>
<th>$C_F$</th>
<th>$C_P$</th>
<th>Rank</th>
<th>$C_D^w$</th>
<th>$C_F$</th>
<th>$C_P$</th>
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<tbody>
<tr>
<td>1</td>
<td>7 (402)</td>
<td>7 (6.3122)</td>
<td>7 (1229.9000)</td>
<td>30</td>
<td>38 (87)</td>
<td>48 (1.5372)</td>
<td>29 (62.1010)</td>
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<tr>
<td>2</td>
<td>20 (379)</td>
<td>3 (5.0308)</td>
<td>3 (812.4900)</td>
<td>31</td>
<td>33 (85)</td>
<td>55 (1.5155)</td>
<td>14 (59.6910)</td>
</tr>
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<td>3</td>
<td>3 (315)</td>
<td>20 (4.7383)</td>
<td>20 (805.8400)</td>
<td>32</td>
<td>23 (81)</td>
<td>38 (1.4535)</td>
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<tr>
<td>4</td>
<td>6 (280)</td>
<td>57 (3.8024)</td>
<td>57 (336.8900)</td>
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</tbody>
</table>

central individual is node 7 shown in Fig. 3 (a), irrespective of which measure is used. What’s more, the results of the least central node are consistent, which is node 28. Fig. 3 (c) shows its ego network. However, some inconsistent results also exist among three different measures. In particular, the rank of node 2 using Physarum centrality is 23rd, which considerably higher than the results of the other two measures (36th and 34th). Although the degree centrality of node 2 is only 72, the edges connecting to it is relatively more critical than other nodes with a bit higher degree centrality. The ego networks of node 7, 2 and 28 is shown in Fig. 3, which is generated by NetDraw network visualization software [20].
5 Conclusion

A central challenge for network analysis is the identification of central nodes within a network. Thus, a number of centrality measures have been proposed. However, the assumption of some measures that information only spreads along shortest paths was regarded unrealistic. By relaxing this assumption, a new centrality measure based on an amoeboid algorithm, *Physarum* centrality, is proposed for weighted networks in this paper. It is considered that not only does information spread along shortest paths, but also competing paths. Our measure adopts an amoeboid algorithm for path finding. As a result, non-competing long edges will be cut off. What’s more, among the reserved edges, shorter edges result in greater flux through them. By means of associating flux with criticality of edges, *Physarum* centrality can be easily calculated. A brief hypothetical example and an application in social network are given to illustrate that our *Physarum* centrality is effective and reasonable.

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References


